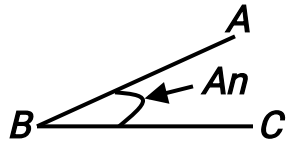


ANGLES

Summary:

1. (i) The space enclosed between the lines **AB** and **BC** is referred to as an angle

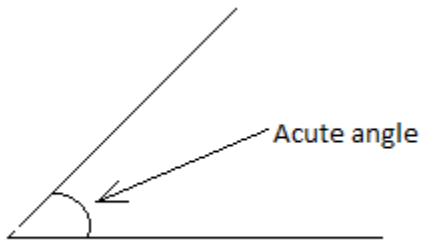


(ii) The above angle is described as $\angle ABC$ or $\angle CBA$ or $\angle B$

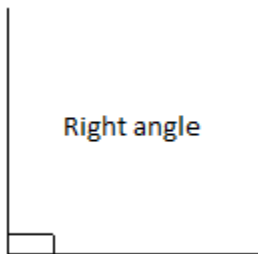
(iii) In geometry, angles are measured in degrees using a protractor

TYPES OF ANGLES.

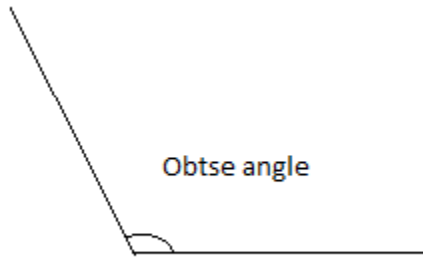
1. An **acute angle** is an angle that is less than 90° .



2. A **right angle** is an angle that is equal to 90° .



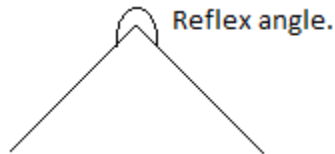
3. An **obtuse angle** is an angle between 90° and 180° .



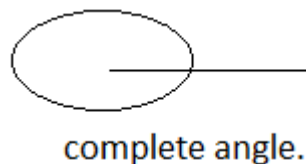
4. A **straight angle** is an angle that is equal to 180° .



5. A **reflex angle** is an angle that lies between 180° and 360° .



6. A **full angle** is an angle that is equal to 360° .

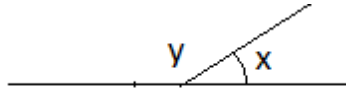


ANGLE RELATIONSHIPS.

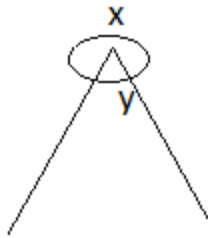
1. **Complementary angles** are two angles that add up to 90° . Thus 40° and 50° are complementary angles since they add up to 90° .
2. Two angles are **supplementary** when they add up to 180° . Thus 30° and 150° are supplementary angles since they add up to 180° .

3. Angles on a line add up to 180° .

Thus $x+y=180^\circ$.

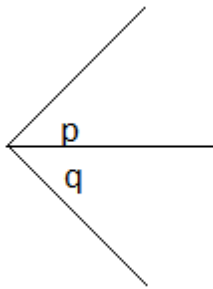


4. Angles around a point add up to 360° .

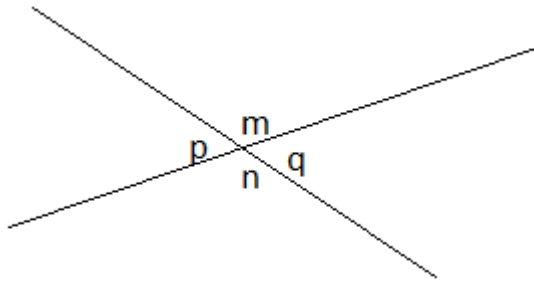


Thus, $x+y=360^\circ$.

5. Two angles next to each other are called **adjacent angles**. Thus, in this example, p and q are adjacent angles.



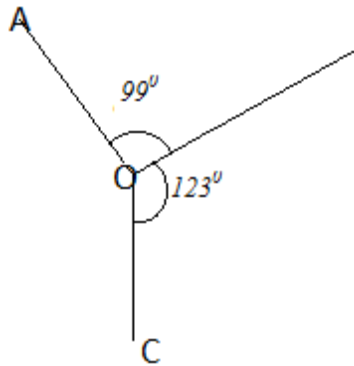
6. **Vertically opposite angles** are angles opposite to each other when two lines cross. In this example, p and q are vertically opposite angles.



Vertically opposite angles are equal. Thus, $p=q$ and $m=n$.

Examples:

1. In the figure below, find the size of angle **AOC**.



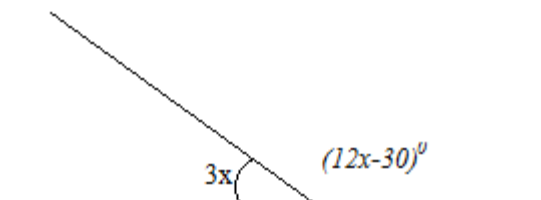
Solution

If $\angle AOC = x$

$$x + 99^\circ + 123^\circ = 360^\circ. \quad (\text{Angles around a point.})$$

$$x = 138^\circ.$$

2. Find the size of each angle in the table below.



Solution

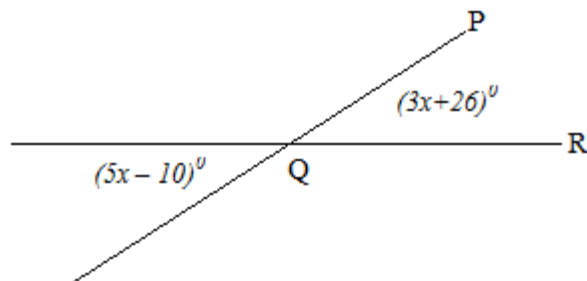
$$3x + (12x - 30)^\circ = 180^\circ.$$

$$x = 14^\circ.$$

$$\text{Angle } 3x = 3(14) = 42^\circ.$$

$$\text{Angle } (12x - 30) = 12(14) - 30 = 138^\circ.$$

3. The figure below shows two intersecting lines.



Find the size of angle **PQR**.

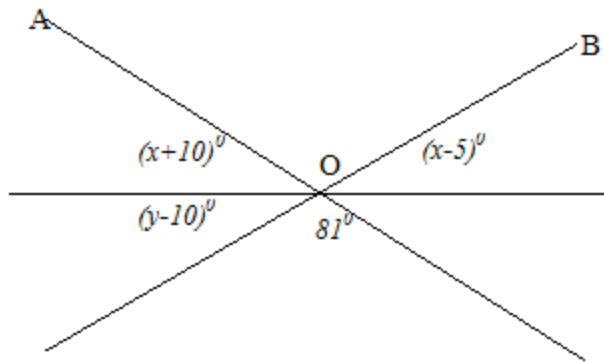
Solution:

$$(5x - 10)^\circ = (3x + 26)^\circ. \quad (\text{Vertically opposite angles})$$

$$x = 18^\circ.$$

$$\text{Angle } PQR = 3(18) + 26 = 80^\circ.$$

4. The figure below shows three intersecting lines.



Find the values of x and y .

Solution:

$$\angle AOC = 81^\circ \quad (\text{Vertically opposite angles})$$

$$(x+10)^\circ + 81^\circ + (x-5)^\circ = 180^\circ \quad (\text{linear angles})$$

$$\therefore x = 47^\circ.$$

$$(x+10)^\circ + (y-10)^\circ + 81^\circ = 180^\circ \quad (\text{linear angles})$$

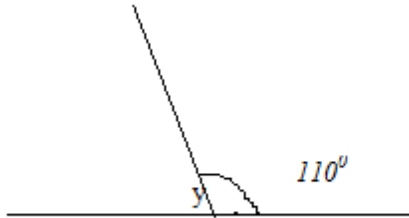
$$(47+10)^\circ + y-10+81=180^\circ$$

$$\therefore y = 52^\circ.$$

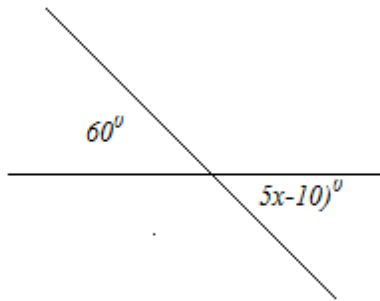
ERR.

1. Find the size of two complementary angles that are such that the size of one of them is four times the size of the other.

2. The ratio of two complementary angles is 1:5. Find the size of each of them.
3. Find the size of an angle that is such that when added on to one – sixth of its complement, the result is 40° .
4. Find the size of angle marked y in the figure below.

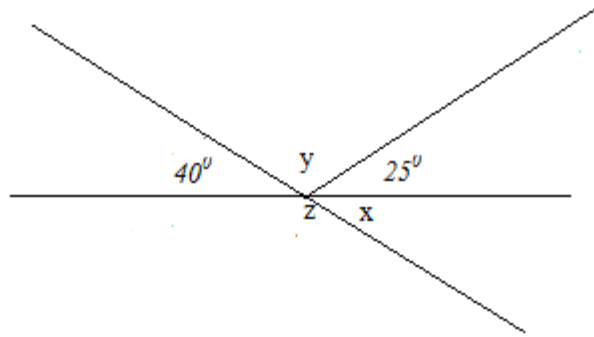


5. Find the value of x for which the angles $(2x+10)^\circ$ and $(130-x)^\circ$ are vertically opposite.
6. The figure below shows two intersecting lines.



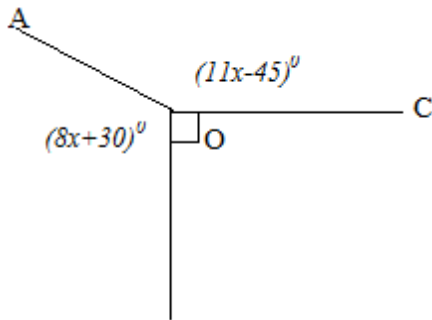
Find the values of x .

7. The figure below shows three intersecting lines.



Find the values of x , y and z .

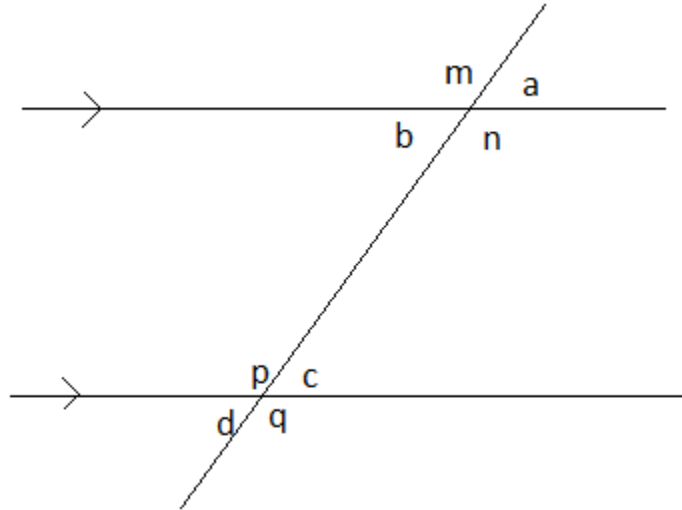
8. In the figure below, find the size of reflex angle AOC .



ANGLES ON A TRANSVERSAL

Summary:

1. A line cutting across parallel lines is called a **transversal**.
2. (i) Below is an illustration of the different angle formed on a transversal.



(ii) The angles in matching corners are called **corresponding angles**.

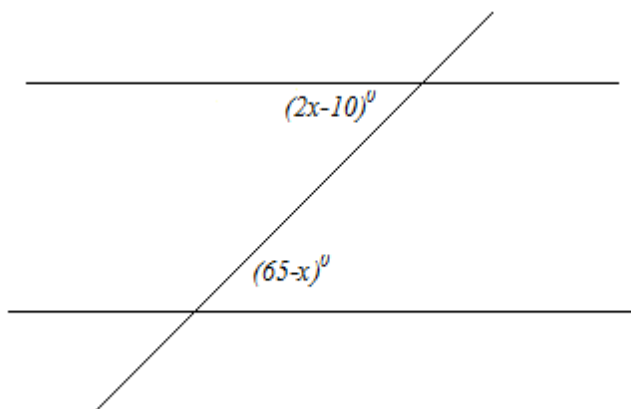
(iii). The interior angles on the opposite sides of a transversal are called **co-interior angles**.

3. The following are the transversal angle properties

- (i) Corresponding angles are equal. Thus $\angle a = \angle c$, $\angle b = \angle d$, $\angle m = \angle p$ and $\angle n = \angle q$
- (ii) Alternate angles are equal. Thus $\angle b = \angle c$ and $\angle n = \angle p$.
- (iii) Co-interior angles add up to 180° , thus $b + p = 180^\circ$

EXAMPLES:

1. The figure below shows parallel lines cut by a transversal.



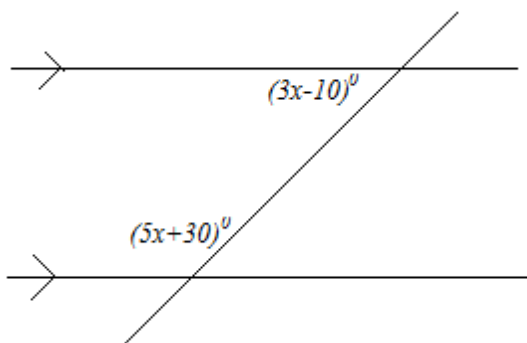
Find the value of x .

Solution

$$(2x-10)^{\circ} = (65-x)^{\circ} \quad (\text{alternate angles})$$

$$\therefore x = 25^{\circ}.$$

2. The figure below shows parallel lines cut by a transversal.



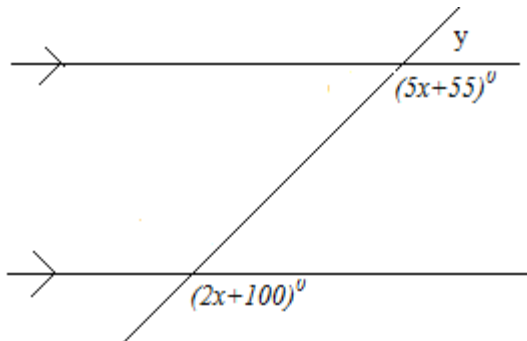
Find the value of x

Solution

$$(3x-10)^{\circ} + (5x+30)^{\circ} = 180^{\circ} \quad (\text{co-interior angles})$$

$$x = 20^{\circ}$$

3. The below shows parallel lines cut by transversal



Find the size of the angle marked y

Solution

$$(5x+55)^{\circ} = (2x+100)^{\circ} \text{ (corresponding angles)}$$

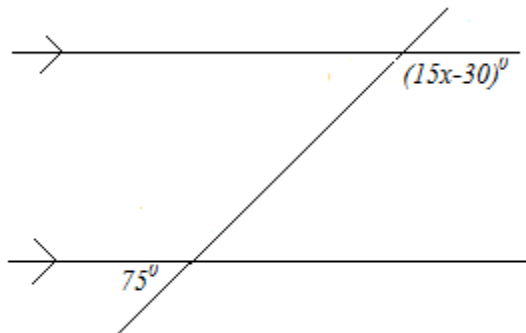
$$\therefore x = 15^{\circ}$$

$$y + (5x+55) = 180$$

$$y + 5(15) + 55 = 180$$

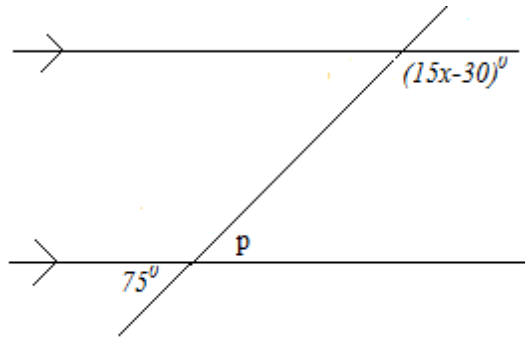
$$\therefore y = 50^{\circ}$$

4. The figure below shows parallel lines cut by a transversal



Find the value of x

Solution

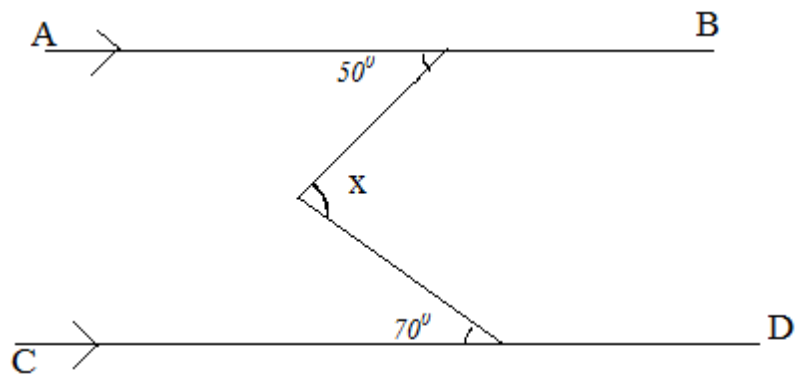


$\angle p = 75^\circ$ (vertically opposite angles)

$$75^\circ + (15x - 30)^\circ = 180^\circ$$

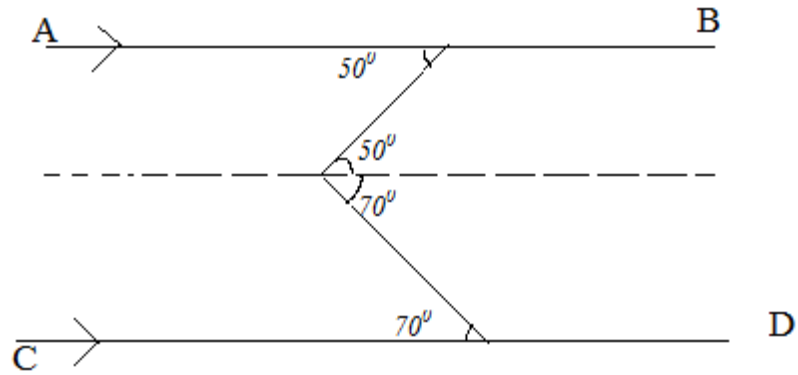
$$\therefore x = 9^\circ.$$

5. The figure below shows parallel lines AB and CD.



Find the size of angle marked x.

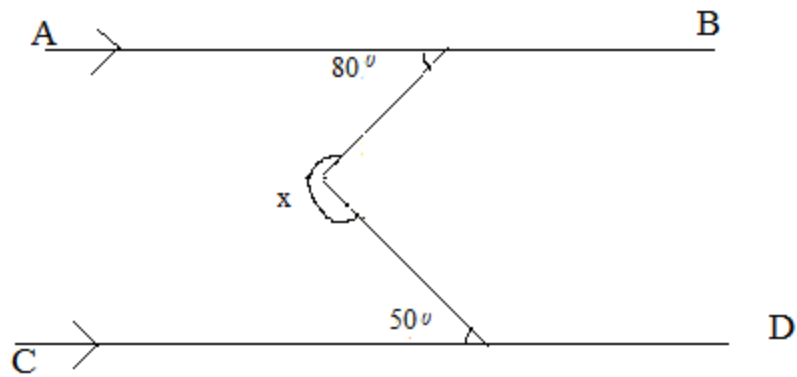
Solution



$$X = 50^\circ + 70^\circ$$

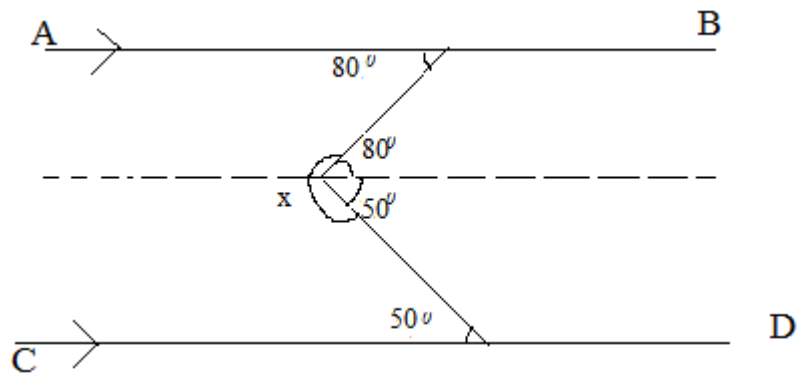
$$X = 120^\circ.$$

6. The figure below shows parallel lines AB and CD.



Find the size of the angle marked x .

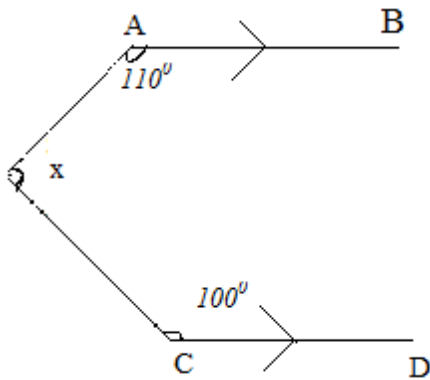
Solution



$$x + 80^\circ + 50^\circ = 360^\circ \text{ (angles around a point)}$$

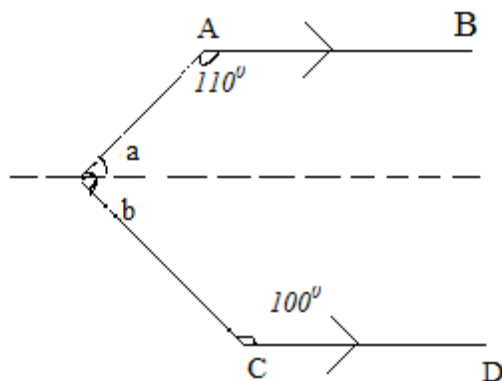
$$\therefore x = 230^\circ.$$

7. The figure below shows parallel lines AB and CD.



Find the size of the angle marked x .

Solution



$$a + 110^\circ = 180^\circ \quad (\text{co-interior angles})$$

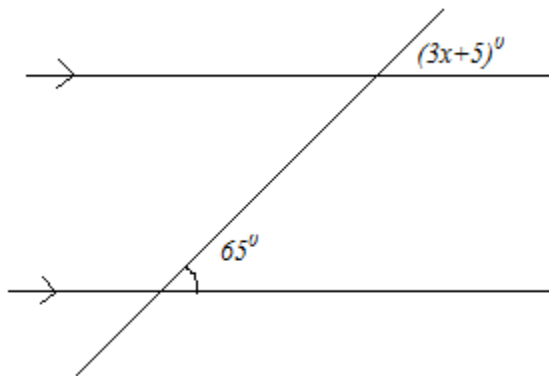
$$\therefore a = 70^\circ$$

$$b + 100^\circ = 180^\circ$$

$$\therefore b = 80^\circ$$

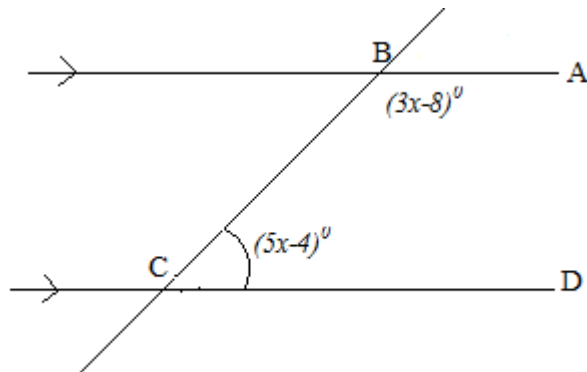
EER

1. The figure below shows parallel lines cut by a transversal.



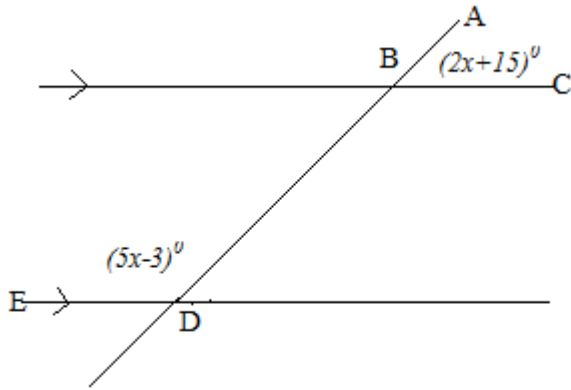
Find the value of x .

2. The figure below shows parallel lines cut by a transversal.



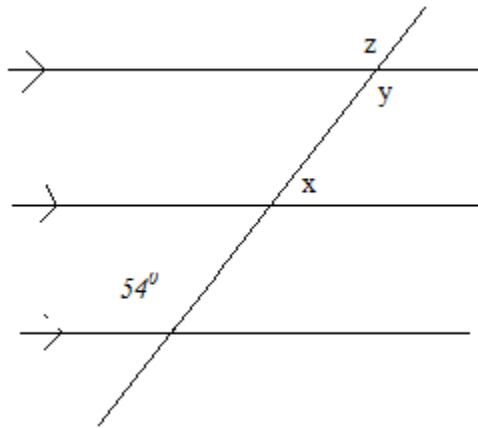
Find the size of angle ABC and angle BCD.

3. The figure below shows parallel lines cut by a transversal.



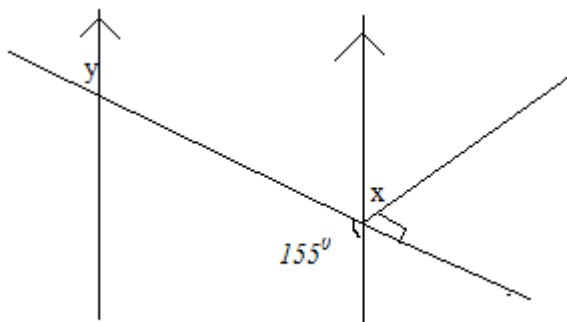
Find the size of angle ABC and angle BDE.

4. The figure below shows parallel lines cut by a transversal.



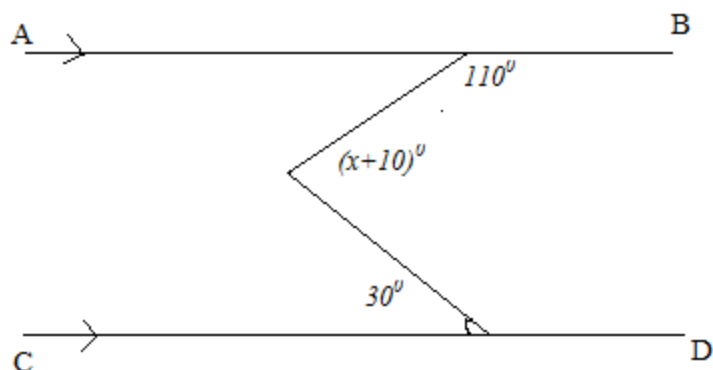
Find the values of x , y and z .

5. The figure below shows parallel lines cut by a transversal.



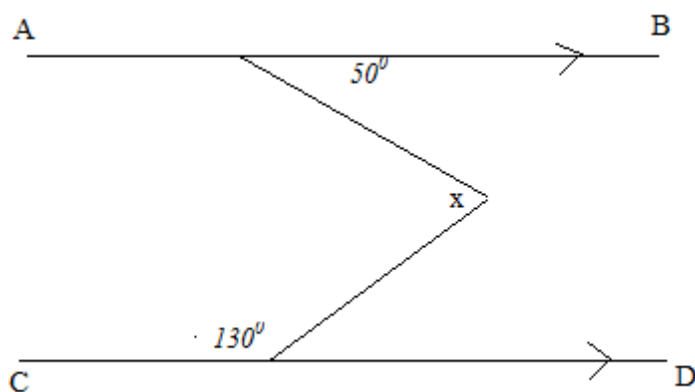
Find the values of x and y .

6. The figure below shows parallel lines AB and CD.



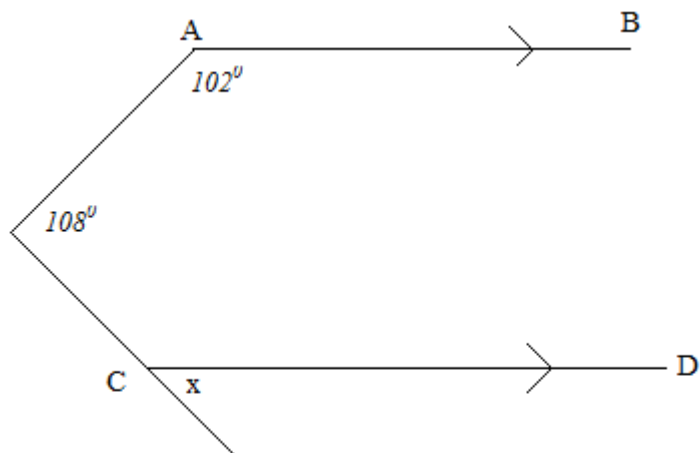
Find the value of x .

7. The figure below shows parallel lines AB and CD.



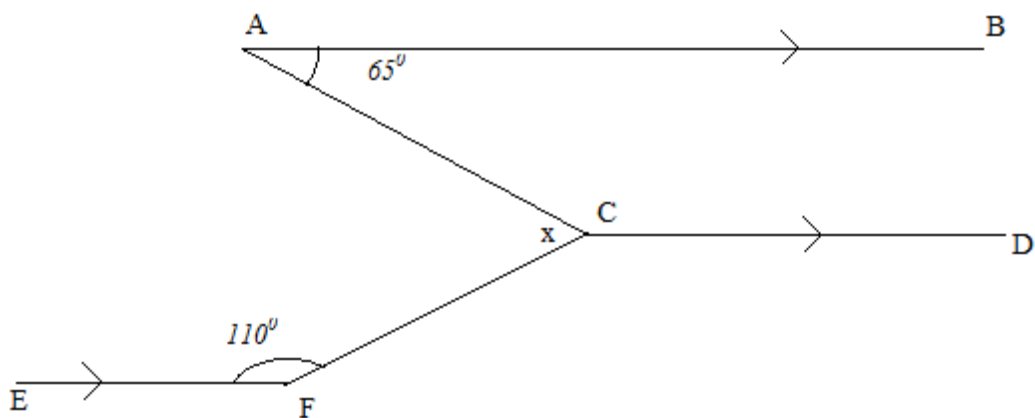
Find the value of x .

8. The figure below shows parallel lines AB and CD.



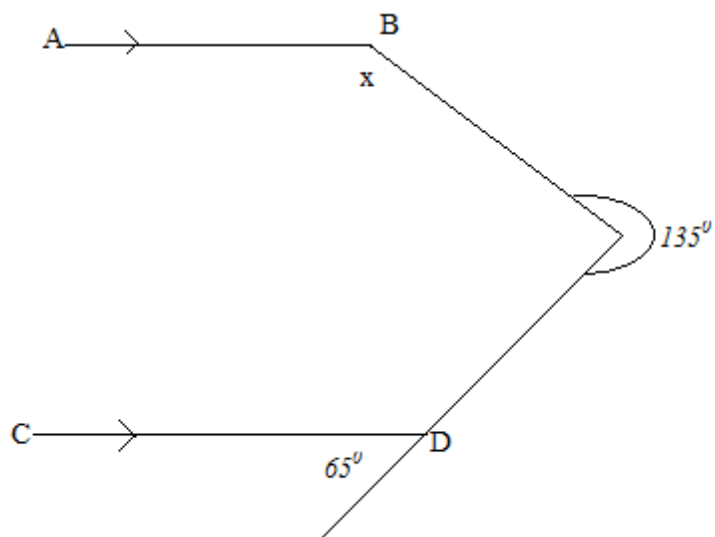
Find the value of x .

9. The figure below shows parallel lines AB, CD and EF.



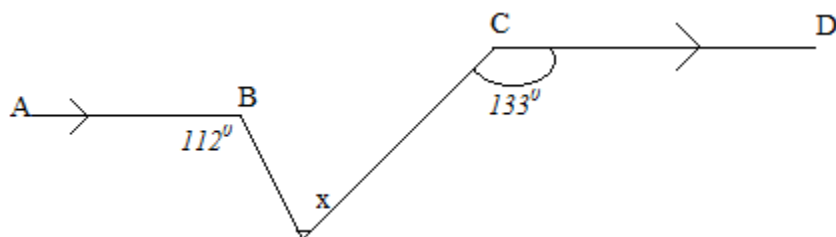
Find the value of x .

10. The figure below shows parallel lines AB and CD.



Find the value of x .

11. The figure below shows parallel lines AB and CD .



Find the value of x .

DIRECTION AND BEARINGS

Summary:

- (i) Bearings are used to show the direction of one point relative to another*
- (ii) The four main directions of a compass are North (**N**), East (**E**), South (**S**) and West (**W**)*
- (iii) A directional compass is as follows:*
- (iv) The bearing to a point is the angle measured in a clockwise direction from the north line.*
- (v) Bearings are stated using three digits. Thus 5° is written as 005°*
- (vi) The north line represents a bearing of 000°*
- (vii) The bearing of **N60°E** means an angle of 60° measured from **N** towards **E***

EXAMPLES:

- 1. The bearing of point **P** from point **Q** is 060° . Find the bearing of **Q** from **P***
- 2. The bearing of point **N** from point **M** is 310° . Find the bearing of **M** from **N***

3. Find the angle between the direction **N70°E** and **S70°W**
4. A boat sails **15km** on a bearing of **000°**. It then sails **8km** due East. Calculate how far it is from the starting point
5. Two ships **P** and **Q** leave port **K** at the same time. **P** sails **9km** on a bearing of **030°** and **Q** sails **12km** on a bearing of **120°**. Calculate how far apart are the ships
6. An observer at point **P** sees an object on a bearing of **100°**. Another observer at point **Q** sees the same object on a bearing of **150°**. Given that the distances of the object from **P** and **Q** are equal, determine the bearing of **P** from **Q**

Soln:

View from P

View from Q

Combined View

If $x + x + 50^\circ = 180^\circ$ "Isosceles triangle"

$$\Rightarrow x = 65^\circ$$

$$\therefore \text{Required bearing} = 150^\circ + x = 150^\circ + 65^\circ = 215^\circ$$

7. A plane flies **300km** from airport **A** to airport **B** on a bearing of **060°**. It then flies **450km** to airport **C** on a bearing of **150°**.

*(a) Use a scale of **1cm** to represent **50km**, make a scale drawing to show the route of the plane.*

*(b) Find the distance and bearing of airport **A** from **C**.*

*(c) If the plane flies directly back to **A** at a speed of 200kmh^{-1} , determine how*

*long it takes to fly back to **A**.*

8. Town *Q* is on a bearing of 060° from Town *P* and **120 km away. Town *R* is on a**

bearing of 130° from *P* and 220° from *Q*.

*(a) By scale drawing show the relative positions of **P**, **Q** and **R**.*

*[Use a scale of **1cm** to represent **20km**]*

(b) Find the distance between:

*(i) **P** and **R***

*(ii) **Q** and **R***

*(c) A plane flies from town **R** on a bearing of 210° at a speed of 150kmh^{-1} .*

*After **40** minutes of flying, the pilot decides to fly directly to town **P**. Find the*

*time it would take to reach **P** and the bearing to which it would fly*

9. A plane flies on a bearing of 060° from airport **A to airport **B** at a steady speed**

of 200kmh^{-1} for **2hours. It then flies on a bearing of 150° to air strip **C** at the**

same original speed for $2\frac{1}{2}$ hours.

(a) Use a scale of **1cm** to represent **50km**, construct a scale drawing to show the route of the plane.

(b) Find the distance and bearing of **A** from **C**.

(c) If the plane flies directly back to **A** at a speed of 200kmh^{-1} , determine how

long it takes to fly back to **A**.

EER:

1. A ship sails **10km** due north and then **24km** due east. Calculate how far it is from the starting point

2. A man walks from town **P** **9 km** due north then **12km** due east to town **Q**. Calculate the distance of **P** from **Q**

3. Find the angle between the direction **N450°E** and **S25°W**

4. A ship sails equal distances due South–East and then due South–West to end up **14km** due South of its starting point. Calculate how long is each part of its journey

5. The bearing of **P** and **Q** from **A** are **200°** and **290°** respectively. Given that distance **AP** = **5.6km** and **AQ** = **4.7km**, find by scale drawing the:

(i) distance **PQ**

(ii) bearing of **P** from **Q**

6. A plane flies from airport **K** due North for **350km** to airport **R**. It then flies on a

bearing of **295°** for **250km** to air strip **N**. From there it flies on a bearing of

090° for **500km** to air strip **M**.

(a) Use a scale of **1cm** to represent **50km**, draw an accurate diagram to show the route of the plane.

(b) Find the distance and bearing of **K** from **M**.

(c) If the plane flies directly back to **K** at a speed of 250kmh^{-1} , determine how

long it takes to fly back to **K**.

7. Town **B** is **100km** away from town **A** on a bearing of **135°**. Town **D** is **124km**

away from town **B** on a bearing of **090°**. Town **C** is **160km** away from town **D**

on a bearing of **030°**.

(a) Use a scale of **1cm** to represent **20km**, draw an accurate diagram to show the

relative positions of the towns.

(b) Find the:

(i) distance and bearing of **C** from **A**

*(ii) distance and bearing of **B** from **C***

8. *A plane flies due west from airport **A** to airport **B** at a steady speed of 280kmh^{-1} for $\frac{3}{4}$ hours. It then alters its course and flies North–West to air strip **C** at 220kmh^{-1} . From there it flies on a bearing of 060° to air strip **D** at 240kmh^{-1} for $1\frac{1}{2}$ hours. The total time of flight between the four air strips is $4\frac{1}{2}$ hours.*

*(a) By scale drawing, determine the distance and bearing of **A** from **D***

*[Use a scale of **1cm** to represent **20km**]*

*(b) Find the average speed for the journey from **A** to **D**.*

*(c) If the plane flies directly back to **A** at a speed of 200kmh^{-1} , determine how*

*long it takes to fly back to **A**.*

9. *A boat sails **450km** from island **M** to island **X** on a bearing of 080° at a speed*

*of 150kmh^{-1} . It then sails on a bearing of 200° to island **Y** at the same original*

*speed for **3** hours. From there it sails at a speed of 200kmh^{-1} to island **Q***

which is west of **M** and **400km** away from it.

(a) Use a scale of **1cm** to represent **50km**, draw an accurate diagram to show the route of the boat.

(b) Find the distance and bearing of **Q** from **Y**.

(b) Find the:

(i) total time the boat takes to cover the whole journey

(ii) average speed of the boat for the whole journey .

10. In a sports field, four points **A**, **B**, **C** and **D** are such that **B** is due south of **A**

and due west of **D**. **AB = 10.8m**, **BD = 18.8m**, **CD = 16.6m**, $\angle BDA = 60^\circ$,

$\angle CDB = 40^\circ$ and $\angle BCD = 80^\circ$.

(a) Use a scale of **1cm** to represent **2m**, draw an accurate diagram to show the

the relative positions of the points.

(b) Find the:

(i) distances **BC** and **AD**

(ii) bearing of **B** from **C**.

(c) If an athlete runs from point **A** through **B**, **C**, **D** and back to **A** in **16 seconds**,

find the athlete's average speed

11. The bearing of tower **A** from point **O** is **060°** and that of tower **B** from **O**, is

200°. Given that distance **OA = 24km**, **OB = 33km** and tower **C** is exactly half

way between towers **A** and **B**,

(a) Use a scale of **1cm** to represent **5km**, draw an accurate diagram to show the

relative positions of the towers.

(b) Find the:

(i) distances **AB** and **OC**

(ii) bearing of **B** from **A**

(iii) bearing of **C** from **O**

(c) Find:

(i) the average speed of the cyclist who takes $2\frac{1}{4}$ hours to travel directly from **A** to **O**

(ii) how long it takes another cyclist to travel from **A** to **B** via **O** at

a

speed of $4 \cdot 5 \text{ kmh}^{-1}$ faster than that of the cyclist in **(c) (i)**

above

12. A plane left airport **K** at **0600** hours and flew on a bearing of **090°** at a speed

of 300 kmh^{-1} . It landed at airport **R** at **0830** hours. At exactly **0900** hours, it

left **R** and flew on a bearing of **340°**, at the same original speed. It then landed

at airport **M** at **1200** hours

(a) Use a scale of **1cm** to represent **100km**, draw an accurate diagram to show the route of the plane.

(b) Find the:

(i) distance of **M** from **K**

(ii) bearing of **K** from **M**

13. A rally car travels from point **R** to point **S** which is **260km** away on a bearing of **060°** from **R**. The car is then set off from **S** at **9:30 am** towards

T at an average speed of 150kmh^{-1} where it is expected to reach at **11:30 am**. After travelling for one hour and twenty minutes, it broke down at

P. The bearing of **T** and **P** from **S** is **300°**.

(a) Using a scale of **1cm:40km**, show positions of points **R, S, P** and **T**

(b) Determine the:

(i) distance from **R** to **P**

(ii) bearing of **P** from **R**

(c) Given that the repair took ten minutes and later the car is set off to complete the journey to **T**. Find the speed at which the car must be driven to reach **T** on time.

EER: S·3 WORK

1. A boat sails **15km** on a bearing of **000°**. It then sails **8km** due East. Calculate the distance and bearing of the ship from its starting point

2. Two ships set off from port **P** at the same time. One ship sails **8km** on a bearing of **030°** to reach point **Q** and the other ship sails **15km** on a bearing

of **120°** to reach point **R**. Calculate the:

(i) distance and bearing of **R** from **Q**

(ii) area of the figure bounded by **PQR**

3. Two ships set off from port **P** at the same time. One ship sails **70km** on a

bearing of **050°** to reach point **Q** and the other ship sails **150km** on a bearing

of **110°** to reach point **R**.

(a) Calculate the:

(i) distance and bearing of **R** from **Q**

(ii) area of the figure bounded by **PQR**

(b) If both ships take **t** hours to reach their destination and the speed of the

faster ship is 60kmh^{-1} , find the:

(i) value of **t**

(ii) speed of the slower ship

4. A man walks from town **P** **9 km** due north then **12km** due east to town **Q**. Calculate the distance and bearing of **P** from **Q**

5. Port **B** is **25 km** east of port **C**. A navigator observes that the bearing of **C**

from

his ship is 310° and that of **B** is 018° .

(a) Calculate the:

(i) distance and bearing of the ship from **B**

(ii) distance and bearing of the ship from **C**

(b) If the ship begins to sail at a speed of 10 kmh^{-1} on the bearing of 240° ,

determine the distance and bearing of the ship from **C** after **48 minutes**.

TRIANGLES

Summary:

1. In any triangle:

- (i) The three angles add up to 180° .
- (ii) The exterior angle is equal to the sum of the two opposite interior angles.
- (iii) The largest angle is always opposite to the longest side.
- (iv) The smallest angle is always opposite to the shortest side.

2. In an equilateral triangle:

- (i) All the three sides are equal in length.
- (ii) The size of each angle is 60° .
- (iii) There are three lines of symmetry.

3. In an isosceles triangle:

- (i) Two of its sides are equal.
- (ii) The angles opposite to the equal sides are equal.
- (iii) There is one line of symmetry.

4. In a scalene triangle:

- (i) All the three sides are un-equal.
- (ii) All the three angles are un-equal.
- (iii) There is no line of symmetry.

5. In a right angled triangle:

- (i) One of the angles is 90° .
- (ii) The three sides are related by Pythagoras property $a^2+b^2=c^2$.

6. In an acute angled triangle, all the angles acute.

7. In an obtuse angled triangle, one of its angles is obtuse.

EXAMPLES:

1. If two angles of a triangle are 45° and 75° , find the size of third angle.

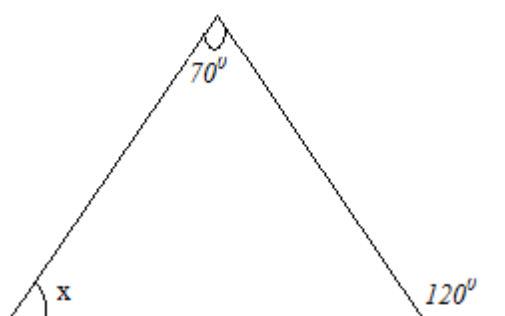
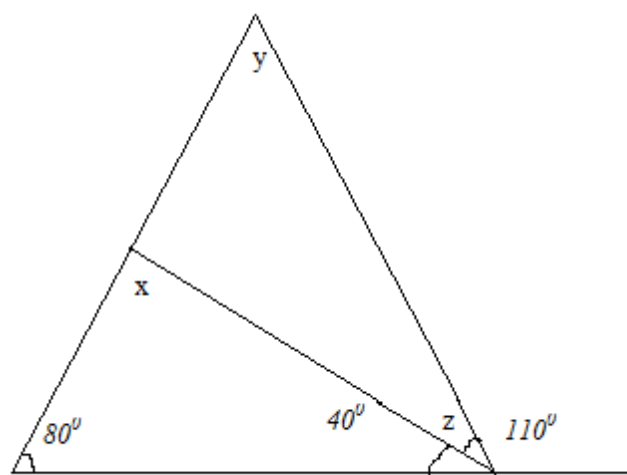
2. The angles of a triangle are $(x+37^\circ)$, $(2x+15^\circ)$ and $(3x+8^\circ)$. Find the:

- (i) value of x .
- (ii) size of each angle.

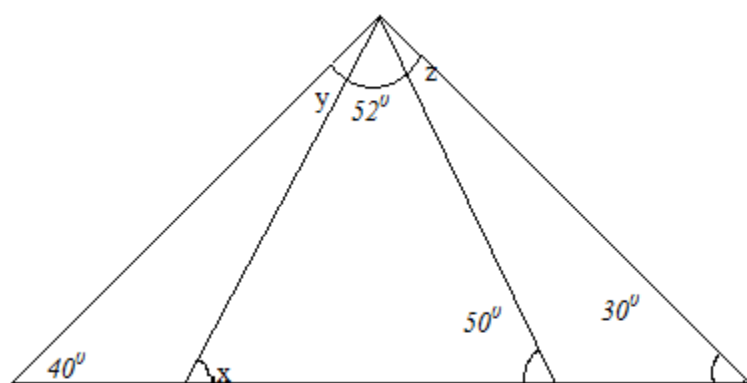
3. If the angles of a triangle are in the ratio 3:4:5, find all the angles.

4. Find the angles marked with letters in the diagrams below:

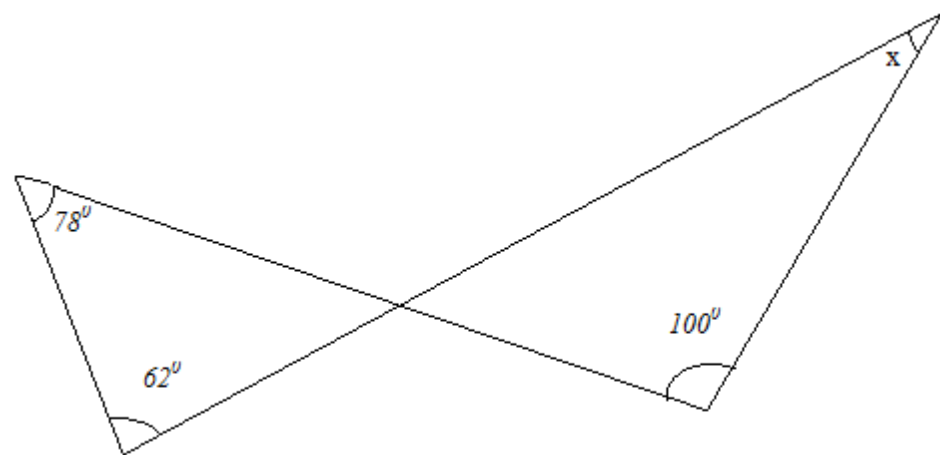
- (i)



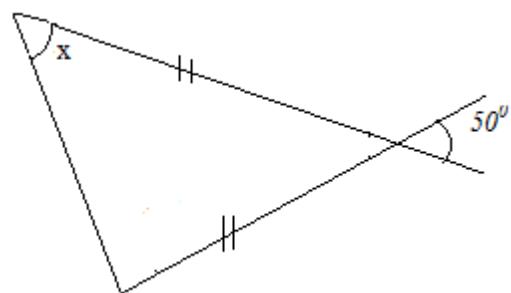
(ii)



(iii)

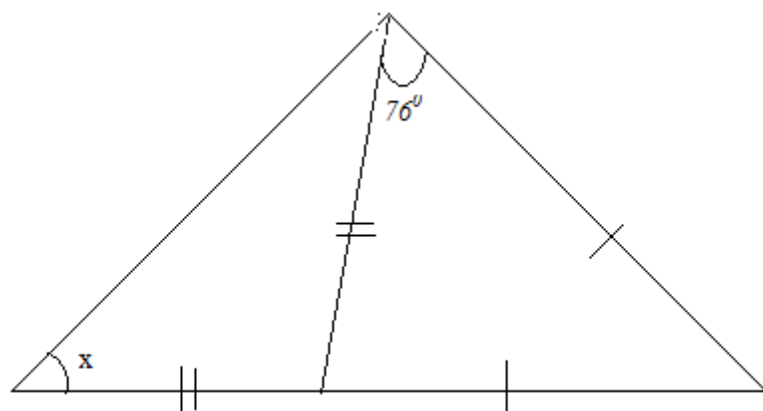


(iv)

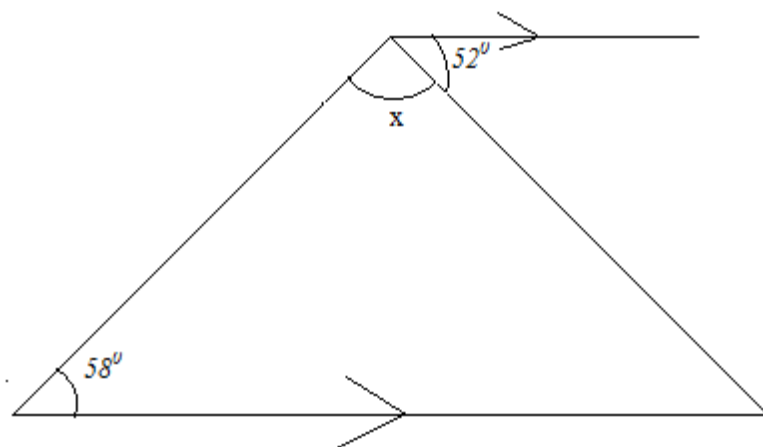
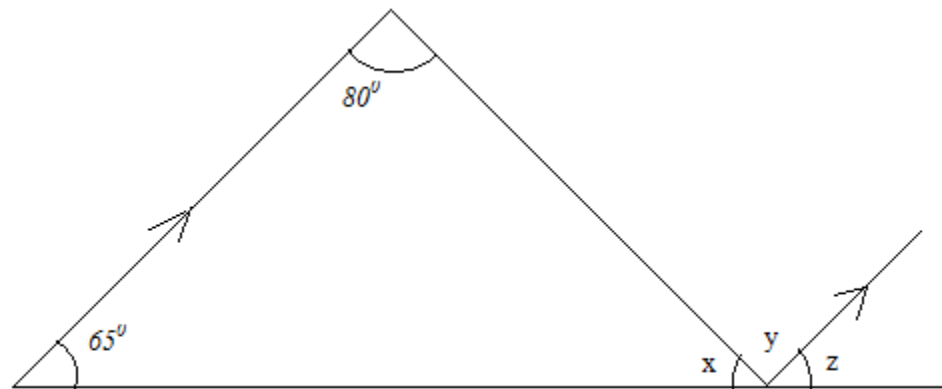


(v)

(vi)

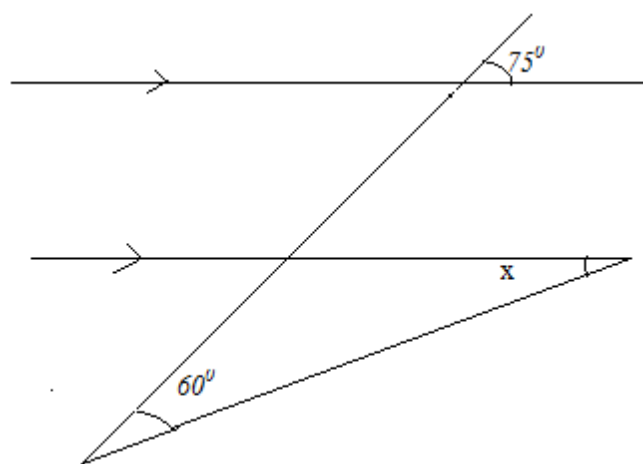
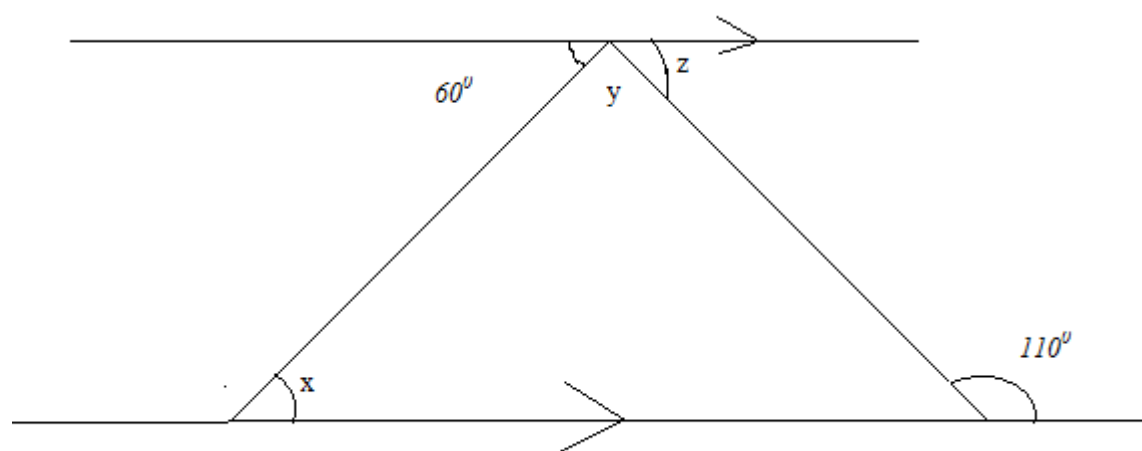


(vii)

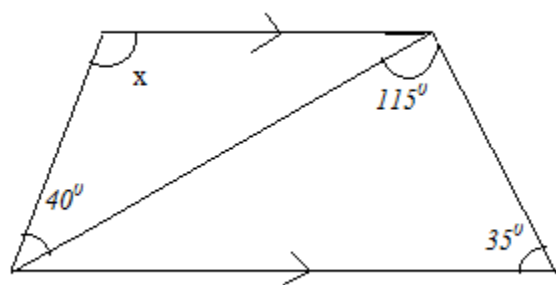


(viii)

(ix)



(x)

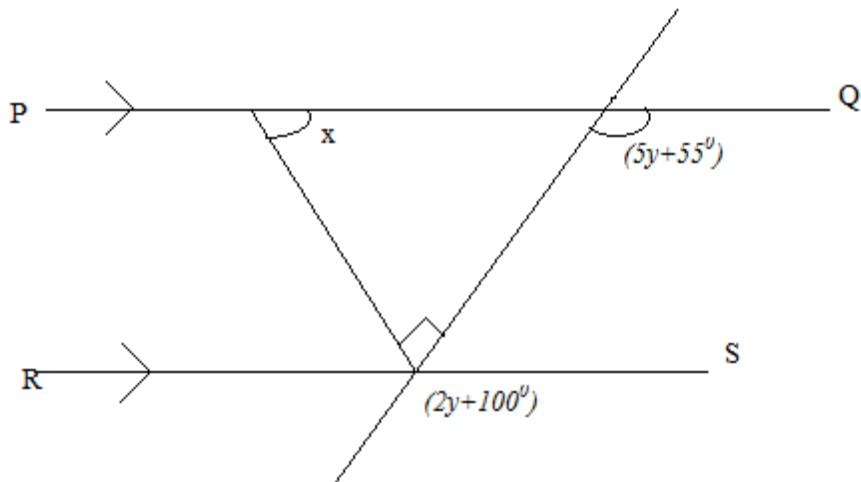


(xi)

5. A ladder of length 13m rests against a vertical wall with its floor 5m away from the wall. Find how far up the wall does the ladder reach.
6. Find the diagonal of a rectangle of length 8cm and width 6cm.
7. A square has diagonals of length 10cm. Find the sides of a square.
8. A cone has base radius 8cm and slant height 17cm. Find its vertical height.
9. Find the length of each side of an equilateral triangle whose height is 15cm.
10. Two buildings 24m apart are 39m and 32m tall. Find the distance between their tops.
11. Find the length of a diagonal of a rectangular box of length 12cm, width 9cm and height 8cm.
12. Find the area of a triangle whose sides are 13cm, 24cm and 13cm.

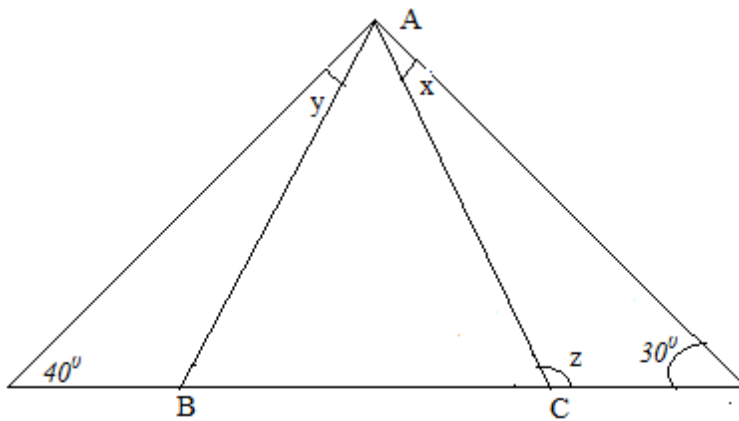
EER

1. The angles of a triangle are $(5x-17^\circ)$, $(3x+20^\circ)$ and $(2x-13^\circ)$. Find the size of each angle.
2. The vertical angle of an isosceles triangle is 58° . Find the base angles.
3. Find the angles of an isosceles triangle if the vertical angle is thrice the base angles.
4. In the diagram below, PQ is parallel to RS.



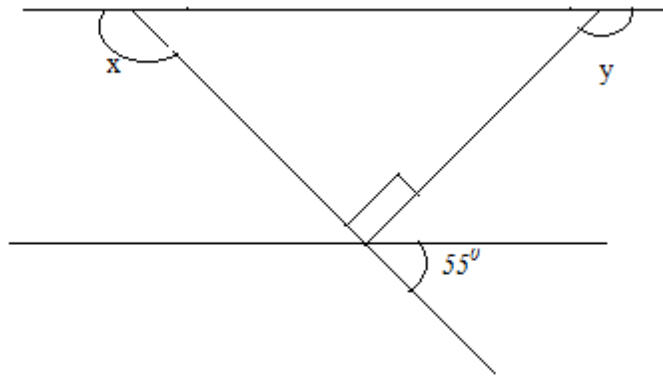
Find the angle marked x .

5. In the diagram, ABC is an equilateral triangle.

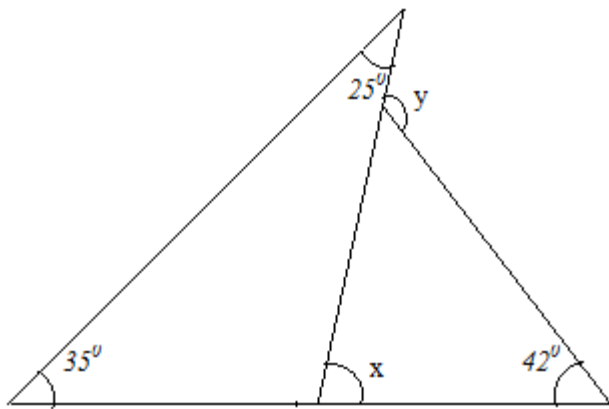


Find the size of angles marked x , y and z .

6. In the figure below, PQ is parallel to RS .

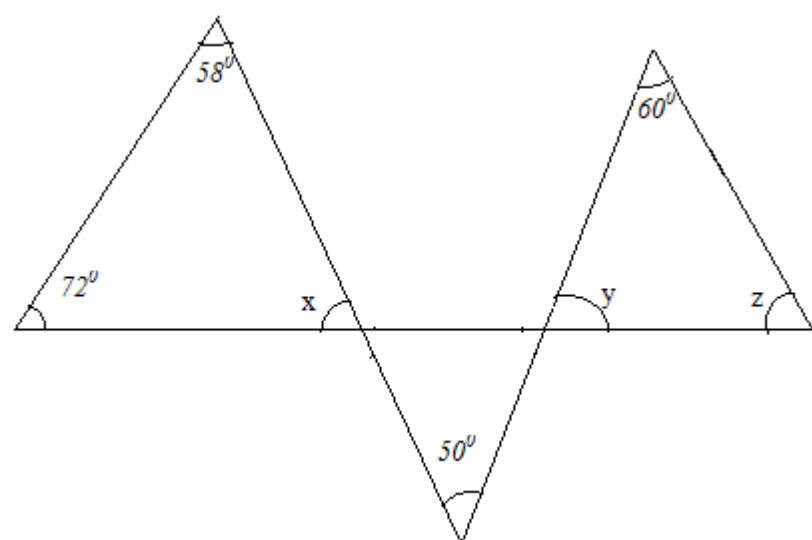


7. Find the angles marked with letters in the figures below:

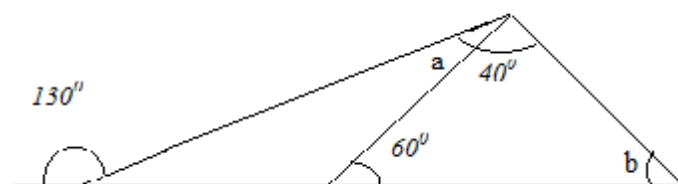


(i)

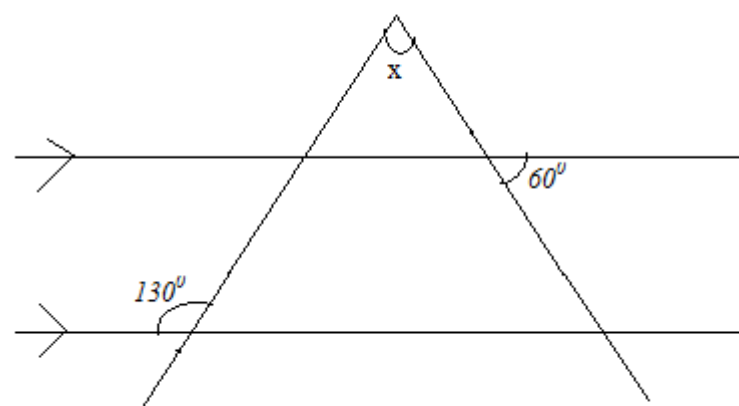
(ii)

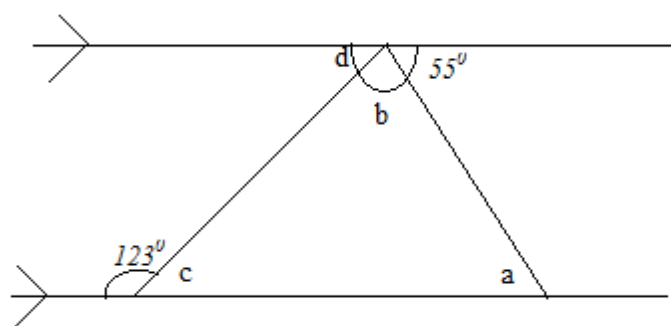


(iii)

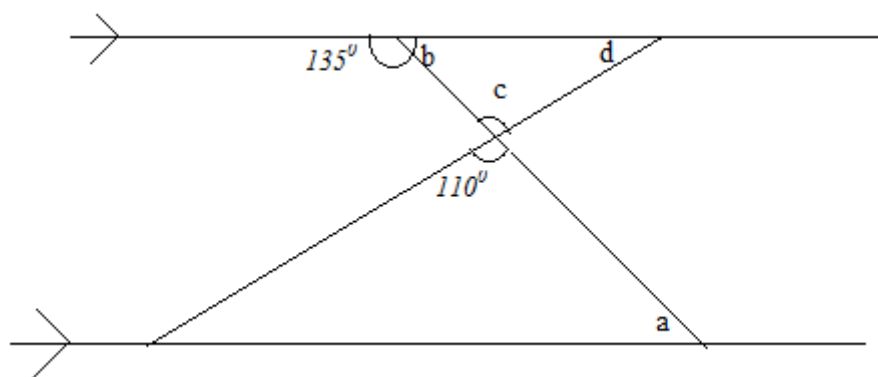


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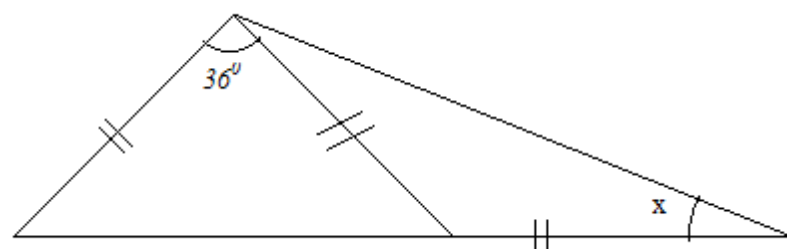
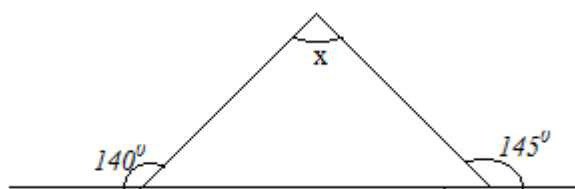




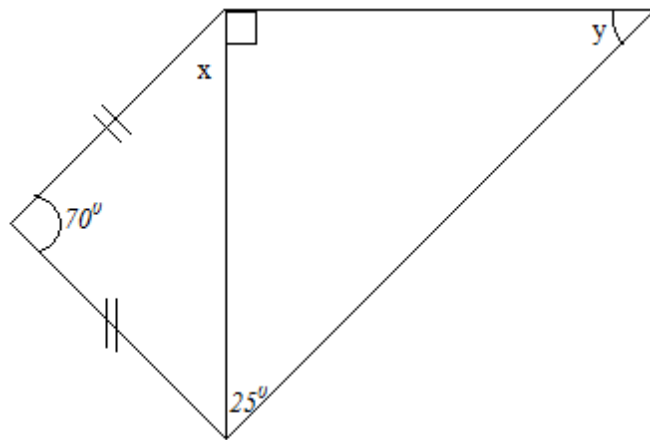
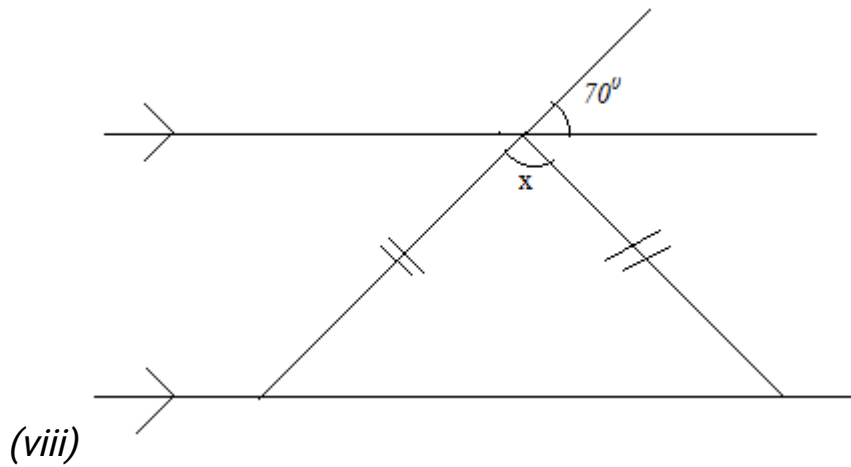
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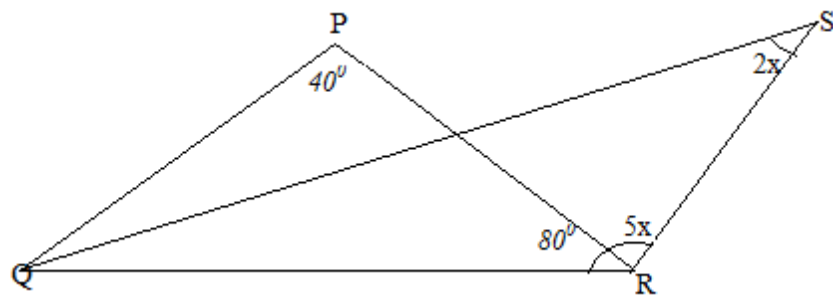
(vi)



(vii)

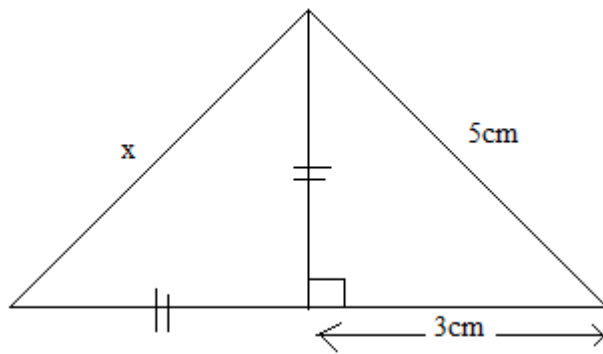


8. In the figure below, QS is the bisector of angle PQR .

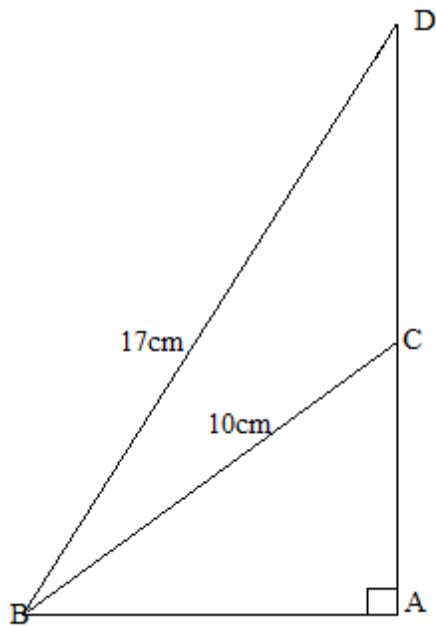


Find the value of x , hence find the angles $2x$ and $5x$.

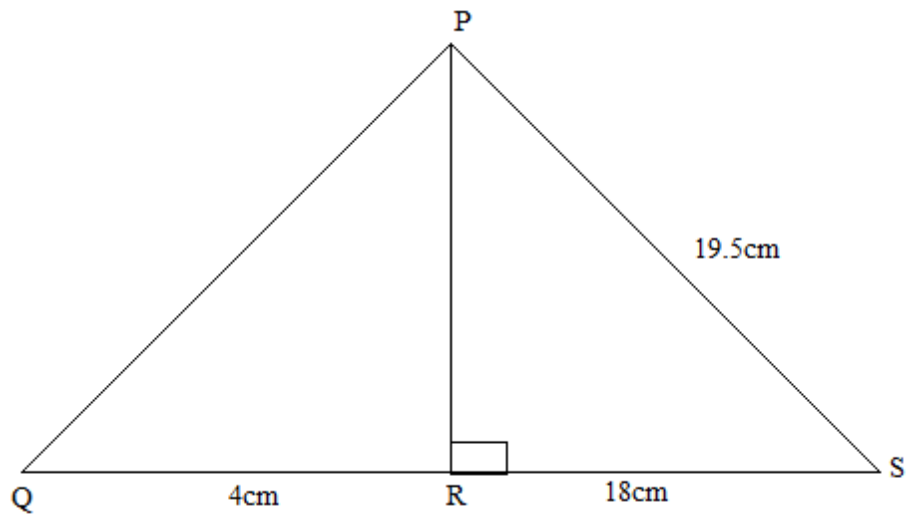
9. In the figure below, find the length of the side marked x .



10. Find the perimeter of a rectangle whose length is 150cm and its diagonal is 170cm long.
11. Find the perimeter of an isosceles triangle whose base is 16cm and its area is 240cm^2 .
12. In the figure below, find the length of CD.



13. In the figure below, find the lengths PR and PQ.

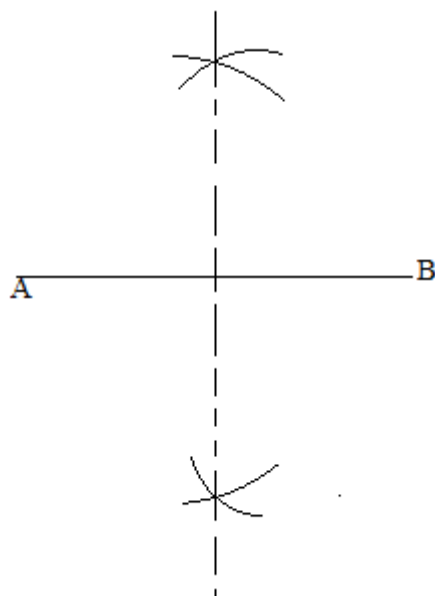


GEOMETRIC CONSTRUCTION

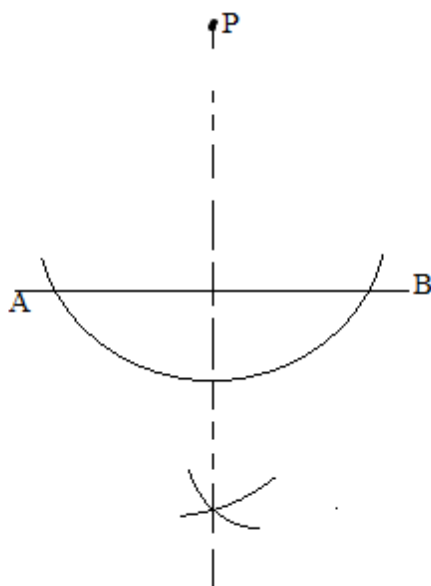
Summary

1. *In construction, we draw angles, lines and shapes using a ruler, pencil and pair of compasses only.*
2. *The angle bisector method can be used to create other angles. Thus, an angle of 45° is obtained by bisecting an angle of 90° .*
3. *The supplementary angle construction method can be used to get obtuse angles. Thus, an angle of 120° is obtained by constructing an angle of 60° .*

4. The construction below shows how to draw a perpendicular bisector of a given line segment AB .



5. The construction below shows how to draw a perpendicular to the line AB from a given external point P .



6. The steps for constructing a circle inscribed in a triangle are as follows:

- (i) Construct angle bisectors of a triangle to meet at the centre of the

circle.

- (ii) Construct a perpendicular from the centre point to one side of the triangle.
- (iii) Place the compass at the centre point and adjust its length up to where the perpendicular crosses the triangle, and then draw the inscribed circle.

7. The steps for circumscribing a circle on a triangle are as follows:

- (i) Construct the perpendicular bisectors of the two sides of the triangle to meet at the centre point of the circle.
- (ii) Place the compass at the centre point and adjust its length up to any vertex of the triangle, then draw the circumscribed circle.

EXAMPLES:

1. Using a pencil, ruler and pair of compasses only, construct the following angles:

- (i) 90° (ii) 45° (iii) $22\frac{1}{2}^{\circ}$ (iv) 135° (v) 60°
- (vi) 30° (vii) 15° (viii) $7\frac{1}{2}^{\circ}$ (ix) 150° (x) 165° .
- (xi) 75° (xii) 82.5° (xiii) 105° .

1. (a) Using a ruler and a pair of compasses only, construct a triangle **ABC** in

which **AB = 6.8cm**, **AC = 5.5cm** and **BC = 4.8cm**. Measure angle **ABC**

(b) Draw a perpendicular from **C** onto **AB** to meet it at **D**. Measure length **CD**

(c) Draw an inscribed circle of triangle **ABC**. Measure the radius of the

circle

(d) Calculate the area enclosed between the inscribed circle and the sides of the

triangle **ABC**.

2. (a) Using a ruler, pencil and a pair of compasses only, construct a triangle **ABC** such that **AB = 8.8cm**, angle **BAC = 75°** and angle **ABC = 45°**. Measure length **AC**

(b) Draw a perpendicular from **C** onto **AB** to meet it at **D**. Measure length **CD**

(c) Draw an inscribed circle of triangle **ABC**. Measure the radius of the circle

(d) Calculate the area enclosed between the inscribed circle and the sides of the

triangle **ABC**.

3. (a) Using a ruler and a pair of compasses only, construct a triangle **ABC** in

which **BC = 7.2cm**, **AC = 8.4cm** and angle **ABC = 75°**. Measure length **AB**

and angle **ACB**

(b) Draw a perpendicular from **A** onto **BC** to meet it at **D**. Measure length **AD**

(c) Draw a circle circumscribing triangle **ABC**. Measure the radius of the

circle

(d) Calculate the area of the segments cut off by triangle **ABC**.

4. (a) Using a ruler and a pair of compasses only, construct a triangle **ABC** in

which **AB = 5.6cm**, **BC = 6.2cm** and angle **ABC = 135°**. Measure length **AC** and angle **BCA**.

(b) Draw a perpendicular from **C** to meet **AB** produced at **D**.

(c) Construct a circle circumscribing triangle **BCD** and state its radius.

(d) Calculate the area of the segments cut off by triangle **BCD**.

5. (a) Using a ruler, pencil and a pair of compasses only, construct a triangle **ABC** such that **AB = 8.6cm**, angle **BAC = 60°** and angle **ABC = 45°**.

(b) **D** is a point on the opposite side of **AB** as **C** such that **AD = BD** and **CD = 11cm**. Draw a circle through the points **A**, **B** and **D**. Measure the:

(i) length of **AC** and angle **ABD**

(i) radius of the circle

Soln

(b) HINT: Triangle **ABD** must be isosceles. Thus point **D** lies on the perpendicular bisector of **AB**

6. Using a ruler and pair of compasses only;

(i) Construct a parallelogram **ABCD** such that **AB = 6cm**, **BC = 4.8cm** and angle **ABC = 150°**.

(ii) Draw a perpendicular from **D** onto **AB** to meet it at **M**. Measure the length **DM**. Hence find the area of the parallelogram **ABCD**.

(iii) Draw a circle through the points **M**, **A** and **D**. Measure the radius of the circle

7. Using a ruler and pair of compasses only, construct:

(i) a quadrilateral **PQRS** such that **QR = 4.5cm**, **RS = 6cm**, **SP = 7.5cm**, **PQ = 10.5cm** and angle **QRS = 45°**.

(ii) point **T** on **RQ** produced such that **PT = ST**. Join the points **P**, **S** and **T**. Measure length **PT** and angle **PTS**.

(iii) a circle through the points **P**, **T** and **R**. Measure the radius of the circle

8. (a) Using a ruler, pencil and a pair of compasses only, construct a triangle **ABC** such that **AB = 6cm**, **AC = 8cm** and angle **BAC = 30°**.

(b) **S** is a point on the opposite side of **AC** as **B** such that **AS = SC** and **BS = 8cm**. Measure length **AS** and angle **ABS**

(c) (i) On the same side of **BS** as **C**, construct the locus of a point **K** such that its

distance from **BS** is the same as the distance of **C** from **BS**

(ii) Given further that angle **BKS = 90°**, find by construction two possible

positions k_1 and k_2 of point **K**. Measure length k_1k_2

Soln

(b) **HINT:** Triangle **ACS** must be isosceles. Thus point **S** lies on the

*perpendicular bisector of **AC***

*(c) (i) HINT: The locus of **K** is a line through **C** and parallel to **BS**.*

*(ii) Since $\angle BKS = 90^\circ$, **K** must lie on the semi-circle with **BS** as diameter*

EER:

1. (a) Using a ruler, pencil and pair of compasses only, construct a triangle **ABC**

*such that **AB** = 7.5cm and **AC** = 11.4cm and angle **ABC** = 120° .*

*(b) Construct a perpendicular from **C** to meet **AB** produced at **D**.*

*(c) Draw a circle circumscribing triangle **BCD**. Hence calculate the area of the*

circle to 2 decimal places.

2. (a) Using a ruler, pencil and a pair of compasses only, construct a triangle **ABC** such that **AB** = 9.7cm, **BC** = 8.6cm and angle **BAC** = 60° .

*(b) **D** is a point on the opposite side of **AB** as **C** such that angle **ABD** = 45°*

*and **CD** = 10.4cm. Draw a circle through the points **B**, **C** and **D**.*

Measure

*length **AC** and the radius of the circle*

3. (a) Using a ruler and a pair of compasses only, construct a triangle **ABC**

in

which $AB = BC = 6.8\text{cm}$ and angle $ABC = 120^\circ$. Measure length AC and angle BCA .

(b) Draw a perpendicular from C to meet AB produced at O .

(c) Construct a circle circumscribing triangle BOC and state its radius.

(d) Calculate the area of the segments cut off by triangle BOC .

44. (a) *Using a ruler and a pair of compasses only, construct a quadrilateral $ABCD$ in*

which $AB = 5\text{cm}$, $BC = 6\text{cm}$, $CD = 9\text{cm}$ and angle $BCD = 135^\circ$

(b) Construct a perpendicular from D to meet BC produced at M .

(c) Construct a circumcircle of triangle CDM and determine the:

(i) length of AD

(i) radius of the circle

45. (a) *Using a ruler and a pair of compasses only, construct a quadrilateral*

$ABCD$ in which $AB = 7\text{cm}$, $BC = 6\text{cm}$, $AD = 5\text{cm}$, angle $BAD = 105^\circ$ and

$ABC = 60^\circ$. Join A to C to form triangle ABC .

(b) Construct an inscribing circle of triangle ABC and determine the:

*(i) length of **AC***

(i) radius of the circle

7. (a) (i) Find the area of a triangle with vertices **P(-2, -2)** **Q(2, 4)** and **R(5, 0)**.

(ii) Construct a circle circumscribing triangle **PQ R**. Hence calculate the

area of the segments cut off by triangle **PQ R**.

4. (a) Using a ruler and a pair of compasses only, construct a quadrilateral

ABCD in which **AB = 5cm**, **BC = 6cm**, **CD = 9cm** and angle **BCD = 135°**

(b) Construct a perpendicular from **D** to meet **BC** produced at **M**.

(c) Construct a circumcircle of triangle **CDM** and determine the:

*(i) length of **AD***

(ii) radius of the circle

45. (a) Using a ruler and a pair of compasses only, construct a quadrilateral **ABCD** in

which **AB = 7cm**, **BC = 6cm**, **AD = 5cm**, angle **BAD = 105°** and

ABC = 60°. Join **A** to **C** to form triangle **ABC**.

(b) Construct an inscribing circle of triangle **ABC** and determine the:

*(i) length of **AC***

(ii) radius of the circle.

POLYGONS

Summary:

1. (i) A polygon is a closed figure with straight sides.

(ii) The table below shows the different polygons.

Number of sides	Polygon name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon

2. In any polygon with n - sides, the following properties apply

(i) Interior angle sum $= (n-2) \times 180^\circ$

(ii) Exterior angle sum $= 360^\circ$.

(iii) Each interior angle + each exterior angle $= 180^\circ$.

(iv) Number of diagonals $= \frac{1}{2} n(n-3)$.

3. In a regular polygon with n - sides, the following properties apply

(i) All the sides and angles are equal.

(ii) Each interior angle = $\frac{\text{interior angle sum}}{\text{Number of angles}}$

$$= \frac{(n-2) \times 180^\circ}{n}$$

(iii) Each exterior angle = $\frac{\text{Exterior angle sum}}{\text{Number of angles}}$

$$= \frac{360^\circ}{n}$$

EXAMPLES:

1. Find the interior angle sum of a decagon.

Solution:

HINT: A decagon has 10 sides.

$$\begin{aligned}\text{Interior angle sum} &= (n-2) \times 180^\circ \\ &= (10-2) \times 180^\circ \\ &= 1440^\circ\end{aligned}$$

2. Find the number of sides of a polygon whose interior angle sum is 900° .

Solution:

$$\begin{aligned}\text{If } (n-2) \times 180^\circ &= 900^\circ \\ \Rightarrow n &= 7.\end{aligned}$$

3. The angles of a hexagon are x , $x+58^\circ$, $x-4^\circ$, 120° , 130° and 140° . Find the value of x .

Solution

$$\text{If angle sum} = (n-2) \times 180^\circ$$

$$\Rightarrow x+x+58+x-4+120+130+140 = (6-2) \times 180^\circ$$

$$3x+444=720^\circ$$

$$\therefore x=92^\circ.$$

4. The angles of a pentagon are in the ratio 3:7:5:4:8. Find the smallest and largest angles of the pentagon.

Solution

$$\text{Angle sum} = (n-2) \times 180^\circ = (5-2) \times 180^\circ = 540^\circ.$$

$$\text{Smallest angle} = \frac{3}{27} \times 540^\circ = 60^\circ$$

$$\text{Largest angle} = \frac{8}{27} \times 540^\circ = 160^\circ$$

5. Four angles of a polygon are 110° each and the remaining angles are 170° each. Find the number of sides of the polygon.

Solution

$$\text{Angle sum} = (n-2) \times 180^\circ$$

$$4(110^\circ) + 170^\circ(n-4) = (n-2) \times 180^\circ$$

$$\therefore n=12.$$

6. Find the size of each interior angle of a regular hexagon.

Solution

$$\begin{aligned}\text{Each interior angle} &= \frac{(n-2) \times 180^\circ}{n} \\ &= \frac{(6-2) \times 180^\circ}{6} \\ &= 120^\circ.\end{aligned}$$

7. Find the number of sides of a regular polygon whose each interior angle is 135° .

Solution

$$\begin{aligned}\text{Each interior angle} &= \frac{(n-2) \times 180^\circ}{n} \\ \frac{(n-2) \times 180^\circ}{n} &= 135^\circ \\ \therefore n &= 8.\end{aligned}$$

8. Find the size of each exterior angle of a regular pentagon.

Solution

$$\begin{aligned}\text{Each exterior angle} &= \frac{360^\circ}{n} \\ &= \frac{360^\circ}{5} \\ &= 72^\circ.\end{aligned}$$

9. Find the number of sides of a regular polygon whose each exterior angle is 40° .

Solution

$$\text{Each exterior angle} = \frac{360^{\circ}}{n}$$

$$\frac{360^{\circ}}{n} = 40^{\circ}$$

$$\therefore n = 9.$$

10. The size of each interior angle of a regular polygon is 4 times the exterior angle. Find the number of sides of the polygon.

Solution

$$\text{If } I = 4E.$$

$$\Rightarrow \frac{(n-2) \times 180^{\circ}}{n} = 4 \left(\frac{360^{\circ}}{n} \right)$$

$$\therefore n = 10.$$

11. The size of each interior angle of a regular polygon is one and a half times the exterior angle. Find the number of sides of the polygon.

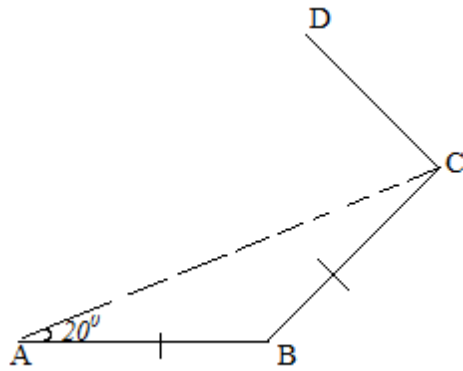
Solution

$$\text{If } I = 1\frac{1}{2}E$$

$$\Rightarrow \frac{(n-2) \times 180^{\circ}}{n} = \frac{3}{2} \left(\frac{360^{\circ}}{n} \right)$$

$$\therefore n = 5.$$

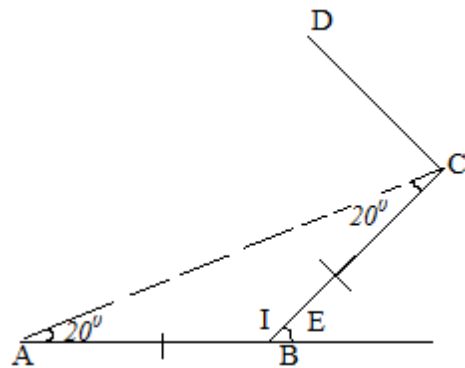
12. The figure ABCD below shows part of the sides of a regular polygon.



Find the:

- (i) Size of each interior and exterior angle of the polygon.
- (ii) Number of sides of the polygon.

Solution



(i) $1 + 20^\circ + 20^\circ = 180^\circ$

$$I=140^{\circ}$$

$$\text{Also, } E=20^{\circ}+20^{\circ}=40^{\circ}.$$

$$(ii) \quad \text{Each exterior angle} = \frac{360^{\circ}}{n}$$

$$\frac{360^{\circ}}{n} = 40^{\circ}$$

$$\therefore n=9.$$

METHOD II

$$\text{Each interior angle} = \frac{(n-2) \times 180^{\circ}}{n}$$

$$\frac{(n-2) \times 180^{\circ}}{n} = 140^{\circ}$$

$$\therefore n=9.$$